A new path toward mathematical fictionalism

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Louis Rouillé

FNRS / ULiège

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Plan

Ontological arguments in general

What does it mean to be ontologically committed?

Commitment to the nonexistence of mathematical objects

Plan

Plan

Ontological arguments in general
- Preliminaries
- Focus on indispensability arguments

What does it mean to be ontologically committed?
- Intellectual dishonesty
- Non-committal strategies
- Counter-IAs

Commitment to the nonexistence of mathematical objects
- A new path toward mathematical fictionalism
- Extraction and abstraction
- Galileo's commitments
- Existence and empriricism

Ontology

- At first approximation, *ontology* is the field of metaphysics concerned with the general question: "What is there?" (Quine 1948)
  - *Metaphorically*, the ontologist's goal is to systematically review the "furniture" / "stuff" reality is made of.
  - *Technically*, the ontologist discusses the validity and scope of ontological *arguments* to be found in the (philosophical) literature.
  - **Csq**: the field of ontology is, *de facto*, controversy-ridden.
- Ontological *arguments* are arguments whose conclusion is an ontological claim.
  - An *ontological claim* is a proposition whose logical form is: "X does (not) exist".
  - An argument is a structured set of propositions: a set of premises + a conclusion which *logically* follows.

Ontological arguments inside and outside the philosophical tradition

- Inside philosophy:
  - Zeno of Elea concludes that "movement does not exist" from a series of paradoxes involving the continuum.
  - Plato in *Republic X* concludes "the immortal soul exists" from the impossibility of its corruption.
  - Anselm in his *Proslogion* demonstrates that "God exists" from God's essential attributes.
- Outside philosophy:
  - Euclid proved that "the greatest prime number does not exist".
  - Historians somehow concluded that "Homer does not exist" and "Jesus of Nazareth did exist".
  - It is generally acknowledged that one can argue from Darwin's theory of evolution to the "nonexistence of God".

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Indispensability arguments...

- With the rise of modern logic, a new kind of ontological argument appeared, so-called *indispensability arguments* (IA):
  - They are based on the technical notion of *quantification* and *reference*.
  - In FOL, both quantification and reference are conceptually linked to existence.
    - *Roughly*: the link between reference and existence is famously fastened in (Russell 1905);
    - that between quantification and existence in (Quine 1939).
    - "Free logicians" accept the former and not the latter (Lambert 1963); "neo-meinongians" reject both (Routley 1966).
- Granted this conceptual link: one can infer *existence* from *truth*, via *ontological commitments*. In its more general form, an IA is:
  1. Such talk appears to be truth-evaluable.
  2. Such talk presupposes (i.e. quantifies over) the existence of such entities.
  3. Therefore, such entities exist.[1]

[1] One can / should distinguish between *local* and *holistic* versions of IA depending on whether the talk is about a theory of everything or not (discussion about the *canonical language*). But this distinction should not affect my argument below, so I put it on the side.

... made famous

- The first neat IA is the Quine-Putnam argument in the philosophy of mathematics (see in particular (Quine 1976) and (Putnam 1971)):
  1. Physical theories are clearly truth-evaluable.
  2. Physical theories clearly quantify over many kinds mathematical entities (numbers, functions, sets, etc.).
  3. Therefore, many mathematical entities exist.
- From then on, many philosophers used or criticised IAs in many different fields.
  - One of the most spectacular use of an IA is (Lewis 1986)'s argument for modal realism.
  - One of the most spectacular criticism of an IA is (Field 1980)'s argument for mathematical fictionalism.

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Interpreting *ontological commitment*

- Why should we accept an IA? Why is an inference from truth to existence valid? Where does the force of the *commitment* come from?
- *Technical reasons*:
  - *Principle of compositionality*: truth presupposes reference, which presupposes existence.
  - *Metaphysical-epistemological considerations*: truth comes with *truth-makers*, and something has to exist in order to *make* a proposition true; truth-makers are also supposed to play an explanatory role (Field 1980).
- An *ethical reason*?
  - In (Putnam 1971: §8) there is a suggestion that accepting an IA is simply a form of "intellectual honesty".
  - I have not seen any serious discussion of this, and so I will toy with this idea a little.

Putnam's "intellectual dishonesty"

- Here is the quote from (Putnam 1971) "VIII. Indispensability Arguments":

> *So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical; therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question. This type of argument stems, of course, from Quine, who has for years stressed both the indispensability of quantification over mathematical entities and the intellectual dishonesty of denying the existence of what one daily presupposes. But indispensability arguments raise a number of questions, some of which I should like briefly to discuss here.*

The core idea

- Consider an intellectually honest person:
  - They are, *by default*, committed to what they say...
  - ... and also to every presupposition accompanying what they say.
  - **ex**: if I told you that I stopped smoking, then my having been a smoker is presupposed; denying this will make me an intellectually dishonest person.
- Making an IA consists in going through what is said *and presupposed* to make explicit one's commitments (*equivalently*: the making explicit one's quantification domain).
  - **Remark**: There's a *Miranda warning* flavour to this way of understanding what ontological commitment means:
    - "You have the right to remain silent. Anything you say can be used against you in court".
    - *Take home message*: "intellectual honesty" is not tied to the locutor's attitude but to the linguistic norms of their utterances.

Newton's *intellectual dishonesty*?

- When reflecting on his "law of universal attraction"

$$F_{A/B} = F_{B/A} = G\frac{M_A.M_B}{d^2}$$

  Newton understood that he was committed to action at a distance.
- He did not quite like it, as he told Bentley in a famous letter from 1692/3:

> *It is inconceivable that inanimate Matter should, without the Mediation of something else, which is not material, operate upon, and affect other matter without mutual Contact... That Gravity should be innate, inherent and essential to Matter, so that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it. Gravity must be caused by an Agent acting constantly according to certain laws; but whether this Agent be material or immaterial, I have left to the Consideration of my readers.*

- He manifestly saw his ontological commitment to action at a distance; did not like it; could see no way around it; and eventually gave up.
- That gave the "hypotheses non fingo"[2] bit from the preface of the 2nd edition of *Principia* in 1713:

  I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction.

  - **Rephrasing**: "hypotheses" (= ontological commitments) are not part of "experimental philosophy" (= the natural science).
  - He *explicitly* says (*inter alia*) that his theory of gravitation should not be judged against the ontological commitment of action at a distance.
  - By Putnam-Quinean standards, Newton is thus intellectually dishonest!

[2] For what it's worth: "fingo" is the latin root for "fiction".

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How to resist an IA? (Yablo 2001)

There is the following predicament. One, we find ourselves uttering sentences that seem on the face of it to be committed to so-and-so's—sentences that could not be true unless so-and-so's existed. But, two, we do not believe that so-and-so's exist. What is someone caught up in The Predicament (as let's call it) supposed to do? The official standard menu of options was given by Quine in Word and Object. Our choices are three:

(1) Show how the commitment can be paraphrased away [...]

(2) Stop uttering the problematic sentences [...]

(3) Give up our resistance to the commitment [...]

Those who reject these options are subjected by Quine to some pretty withering criticism: "I deplore the philosophical double talk, which would repudiate an ontology while simultaneously enjoying its benefits" (242).

[...] It appears then that Quine recognizes a fourth way of dealing with The Predicament. Someone whose sentences are committed to so-and-so's need not share in the commitment if

(4) the sentences are advanced in a fictional or make-believe spirit.

To have a name for this fourth option, let us call it fictionalism. There are a number of versions of fictionalism, according to the various accounts one might give of "advancing in a fictional spirit."

Honest doubts about ontological commitments

- It is not true that "*anything* I say can be used against me".
- In fact, I am not committed when I am not serious:
  - *As-if* talk:
    - Fictional talk;
    - Reporting someone's beliefs;
    - Counterfactual talk ...
  - Figurative talk:
    - "You should hold your horses!"
    - "Einstein was a biographer's dream"
  - Metaphorical talk:
    - "It is the east, and Juliet is the sun"
    - "Tibet is in the roof of the world"
  - ...
- *The fictionalist line*: argue that your favourite piece of discourse is *non-serious* to dodge the corresponding IA, e.g.:
  - (Mac Lane 1986)'s "mythical Platonism" is a kind of "as if" fictionalism.
  - (Yablo 2001) argues for "figuralism".

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Negation of IAs

- So far we have:
  - *commitment to existence* arguments: i.e. indispensability arguments.
  - *non-commitment to existence* arguments: i.e. means to resist an indispensability argument.[3] (Yablo 2001).
- *Formally*, there is a third possibility, viz. *commitment to nonexistence*.
  - Supposing commitment operates on an existence predicate:

$$C_A\mathcal{E}x \qquad \neg C_A\mathcal{E}x \qquad C_A\neg\mathcal{E}x$$

- We can thus construct "counter-indispensability arguments": going from truthful talk to *nonexistence* claims.[4]

[3] For once, French is better than English on this distinction: there is a nice symmetry between "engagement et dégagement ontologique".

[4] **Terminological remarks**: "Dispensability arguments" would have been a nice term, but it really does not say what I want to say. "Tinological commitment" would also have been great if (Aubenque 1991)'s neologism for a "science of non-being" had been taken up. As a fun name, I used to call them "don't say 'no', you smiled!" arguments as a reference to the *Children of Paradise*.

Examples

- Suppose you say to Bernard-Henry Levi: "Jean-Baptiste Botul is a hoax"[5]
  - That someone is thereby committed to Botul's nonexistence.
  - Denying this commitment would be intellectually dishonest.
- There are many other ways of "characterising nonexistence" (Kroon 1996), or many "nonexistence entailing predicates"[6]:
  - talk of *imaginary* friends; figment of the *imagination*; ...
  - talk of *failed* posits; *hypothetical* entities; *theoretical* artifact...
  - talk of *deliberate myths*; *mythical* beings; *useful fiction*; ...
  - talk of *purely fictional* characters, events, persons, places ...
  - talk of *trick of light*; *illusory* entity; ...
  - talk of *virtual* persons, objects, events ...

[5] This statement is (seriously) true. See the wikipedia page for more info, and "You Kant make this up" for a BHL's reaction.

[6] The expression comes from Dolf Rami's DFG project "quantifications, existence-entailing predicates and existential import".

Lessons for ontology

- Here is a counter-IA (from *truth* to *nonexistence*):
  1. (Seriously) My son has an imaginary friend.
  2. Given the meaning of *imaginary*, I am committed to the nonexistence of my son's imaginary friend.
  3. My son's imaginary friend *cannot* be part of my ontology.
- **Methodological claim**: the ontologist's job should be to make explicit *all* our commitments.[7]
  - In order to answer the ontological question "What is there?", one should look at one's commitments to existence *and* nonexistence.
  - Many things we are *prima facie* not committed to either way.
  - Arguably, some things we are sometimes committed to both ways: these breed controversies...
- **Csq**: The controversial part of ontology can thus be seen as an arbitration work, *given* conflicting commitments.

[7] *Catch-phrase*: honesty goes both ways.

Plan

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Dialectics

- Mathematical fictionalism is a sophisticated non-commitment position toward mathematical entities (Linnebo 2018).
  - Its aim is to show that IAs (for mathelatical realism) *do not work*.
  - But it is compatible with a (more sophisticated) version of realism (to be argued for without an IA).
- **Alternatively**, there is a *stronger* version of fictionalism, according to which we are committed to the nonexistence of mathematical entities.
  - That version of fictionalism is, *stricto sensu*, an *anti*-realism:
  - It is incompatible with *any* form of realism.
- **To do**: construct a counter-IA argument, concluding that mathematical entities do not exist.
  - In the remainder, I will explore the following idea:[8]
    - the origin of mathematical entities is mathematical *practice*;
    - in such practice, there is a useful distinction between *extraction* and *abstraction* (Mac Lane 1986);
    - *extraction* commits you to the nonexistence of the extracted object.

[8] Thanks to Yacin Hamami for guiding me on this.

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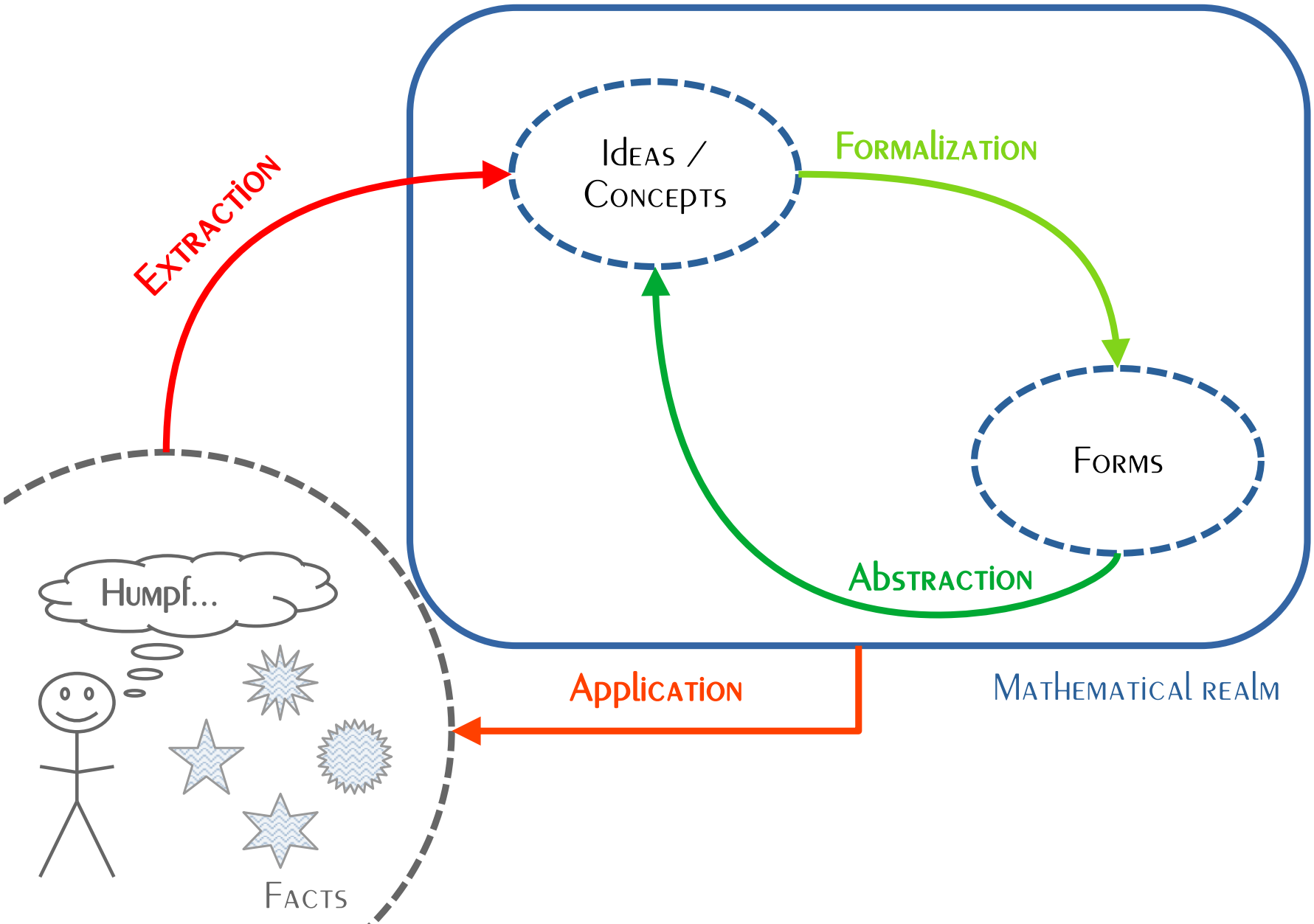
Mathematical extraction and abstraction (Mac Lane 1986: 418; 436)

I assert that subjects of Mathematics are extracted from the environment; that is, from activities, phenomena, or science – and that they are then later applied to that – or other-environments. Thus number theory is "extracted" from the activity of counting, and geometry is extracted from motion and shaping. The exact mechanism of this "extraction" has not been described in detail here; it will clearly vary considerably from case to case. I have deliberately chosen this work "extraction" to be close to the more familiar word "abstraction" – and with the intent that the Mathematical subject resulting from an extraction is indeed abstract. Mathematics is not "about" human activity, phenomena, or science. It is about the extractions and formalization of ideas – and their manifold consequences. [...]

An "abstraction" is intended to pick out certain central aspects of the prior instances, and to free them from aspects extraneous to the purpose at hand. Thus abstraction is likely to lead to the description and analysis of new and more austere or more "abstract" mathematical concepts. We will describe some types of abstraction under headings "abstraction by deletion", "abstraction by analogy", and "abstraction by shift of attention".


Extraction
Ideas / Concepts
Formalization
Forms
Abstraction
Application
Mathematical realm
Humpf...
Facts

Focus on extraction in geometry

- Plane geometry:
  - *Original practices*: motion (falling objects for vertical lines, water waves for circles, ...); construction (column for vertical, triangle for steadiness, ...); drawing/mapping (reproducing shape, scaling, lines of sight, ...); ...
  - *Elementary objects*: points, lines, circles (and constructions thereof) in a plane.
  - The elementary objects are somehow "paradoxical":
    - Points can be located but they have no extension.
    - Lines have only one dimension.
    - Planes are flat but have no thickness.
- By *extracting* the elementary objects of plane geometry, one is thereby committed to talking about something else than reality.

> *It is impossible in practice to produce a plane surface which is truly plane; or to make a spherical surface which is so in reality. Perfectly rigid bodies do not, and cannot, exist in rerum natura; nor can perfect elastic bodies; and it is not possible to make an absolutely correct measurement. Perfection is not of this world: no doubt we can approach it, but we cannot attain it. Between empirical facts and theoretical concept there remains, and will always remain, a gap that cannot be bridged. (Koyré 1992: 45)*

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On "ideal conditions"

- **ex**$_1$: Galileo invented the concept of a "frictionless plane" to predict the motion of an object down an inclined plane.
  - *Notion*: neglecting part of the phenomenon to better model reality.
  - *Pb*: how come frictionless planes have a great explanatory and design value?
  - Galileo's notion *presupposes* nonexistence: It is *because* he understood that there was friction in reality that he could *imagine* a world in which there was no friction (Koyré 1992).[9]
- **ex**$_2$: Galileo's "inertial frame of reference" presupposes *rectilinear motion*. However (Galileo *Dialogo, I*, p. 43):

> *Rectilinear motion is something which, to speak truly, does not happen in the World. Rectilinear motion cannot happen in Nature. Indeed, it is essential for rectilinear motion to be infinite and, while the line is infinite and indeterminate, it is impossible in principle for a moving object to move along a line, for it is impossible for such an object to move toward an impossible end, since there is no end in the infinite.*

[9] The structure of Galileo's theory is counterfactual: "if frictionless planes existed, then motion would follow these equations".

Plan

Toward a practice-based empiricism

- To make a counter-IA based on Mac Lane's notion of *extraction*, there needs to be a further argument according to which *existence* is on the empirical side (and not in the mathematical realm).
  - **Indeed**: *Extraction* is compatible with *idealism* (or "strong platonism"), according to which the empirical and mathematical realm are ontologically distinct, but *existence* is wholly on the mathematical side.[10]
  - *Extraction* is incompatible with *realism* (or "weak platonism"), according to which mathematical entities exist alongside the empirical.[11]
- So given an independent argument in favour of empiricism (or against idealism), we have a new path toward mathematical fictionalism.
  - This new path is strongly *anti-realist*, by committing oneself to the *nonexistence* of mathematical entities.
  - It can be seen as hard-core empiricism, based on a special focus on mathematical *practices*.

[10] Which is close to Plato's platonism about Forms.

[11] Which is the typical conclusion of an IA.

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